MEASURING TIME BETWEEN PEAKS IN HELICOPTER CLASSIFICATION USING CONTINUOUS WAVELET TRANSFORM

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ABSTRACT

Helicopter classification capability is a desirable feature on air defense radars, as a way of distinguishing friends of foes. However, it is still a challenging task for any radar system. The classical method for helicopter classification is the L/N-quotient method, where two signal features are measured: time between peaks and blade tip velocity. Unfortunately, these features are difficult to measure on actual target echoes with high noise levels. This paper shows the difficulties for measuring these features and presents a way of measuring time between peaks using Continuous Wavelet Transform (CWT).

1. INTRODUCTION

Distinguishing targets as friends or foes is a very important capability in any radar based air defense system, in order to avoid friendly fire. Helicopter classification is a great tool to evaluate if a target represents any kind of danger.

Radar based helicopter classification is essentially a pattern recognition problem and, as so, is strongly dependent on signal features extraction. The most important features used nowadays in order to perform helicopter classification are presented in [1]. They are given by measuring the time between successive peaks in time domain and the blade tip velocity in frequency domain.

A better way to measure both features is presented in [2], where the blade tip is measured by a moving windows detection algorithm and the time between peaks is measured by a peak detector procedure after incoherent integration of samples obtained from several successive dwells. However, these features are still very difficult to measure in signals with low signal-to-noise conditions.

An alternative to measure these features is the use of time-frequency analysis techniques. This kind of analysis was first shown in [3], where spectrogram STFT is used to aid in feature extraction. In that work, however, the helicopter fuselage echo is ignored and, due to the STFT's poor resolution in time, the time between peaks is measured in time domain and the STFT is only used to find the number of blades in the helicopter main rotor, which limits the time-frequency analysis tool.

This work presents a way of measuring the time between successive blade flashes using CWT.

2. SIGNAL MODELING

A classical signal modeling for representing helicopter echoes is presented in [4]. This model is based on the concept that the helicopter echo is formed by the backscatter of the followings components:

- Helicopter fuselage echo: similar to a fixed-wing aircraft echo, with high radar cross section, when compared to other helicopter parts, and poor Doppler signature, that is usually not very useful for distinguishing helicopters.
- Helicopter main rotor blades echo: consists of a few blades rotating on angular velocities dependent on the helicopter model. Those blades have high RCS when placed on a perpendicular position relative to the radar line-of-sight, and low RCS in other positions. This produces peaks in time domain, due to the passage of the blades by the position of high RCS.
- Helicopter tail rotor: it has a composition similar to that of the main rotor, but with much smaller RCS, faster rotation rate and with a position relative to the radar much more instable, being hidden by the fuselage in several situations.

Taking those considerations into account, we notice that the most important component to perform helicopter classification is the main rotor blades echo. Then, only this component is modeled.

This model is given by

$$s_j(t) = \sum_{i=0}^{N-1} e^{j\left[\omega_0 t - \frac{2\omega_0}{c}(R - vt) + \Psi_i\right]} \operatorname{sinc}[\Psi_i] \qquad (1)$$

where $j = \sqrt{-1}$ and

$$\Psi_i(t) = \left(\frac{\omega_0 L}{c}\right) \operatorname{sen}\left(\omega_{rot} t + \theta_0 + 2\pi \frac{i-1}{N}\right) \cos\beta \qquad (2)$$

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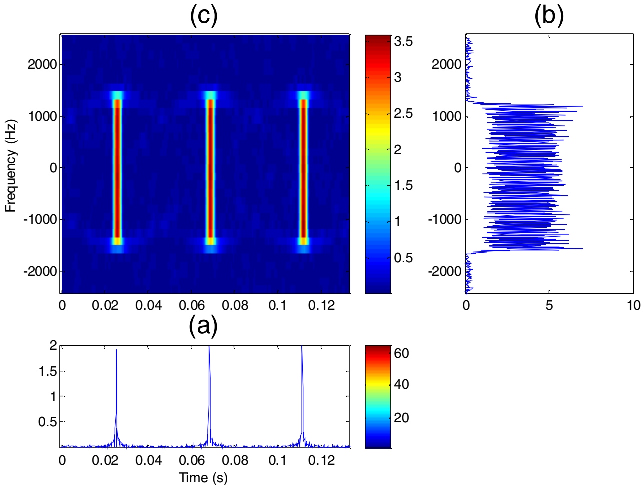

Figure 1 – Classic model for helicopter echo: (a) signal in time domain; (b) FFT spectrum of the signal; (c) STFT spectrogram

where N is the number of blades, L is the blade length, R is the distance between helicopter and radar, v is the helicopter radial velocity, ω_{rot} is the blade rotation angular frequency, θ_0 is the blade initial angular position and β is the helicopter elevation angle relative to radar position. ω_0 is the radar transmission angular frequency and c is the velocity of light.

The result is the signal presented in Figure1. However, the echo of a real helicopter has the strong influence of the fuselage echo. Besides, other influences are strongly present in a real helicopter echo and they have to be taken into account, such as antenna lobe influence.

A more complete model for the signal produced by radar echo of an helicopter is presented in [5]. That model can be described as:

$$s(t) = a_s(t)\left(c_s s_s(t) + c_j s_j(t)\right) + c_n s_n(t) \qquad (3)$$

where:

- s_s is a model of the helicopter fuselage echo, s_n is a narrowband white noise, and s_j is the model of the main rotor presented in [4] and given by equations (2) and (3).
- c_s, c_j and c_n are the normalized coefficients of the fuselage echo, main rotor echo and noise, respectively.
- a_s is the integrated influence of the air defense radar system on the target echoes, and can be given by

$$a_s(t) = a_T(t)a_A(t)a_R(t) \qquad (4)$$

  where a_T is a model of the transmitted radar signal, a_A is a model of antenna scanning, and a_R is a model of radar linear receiver.

  The result is the signal shown in Figure 2, where it can be seen the helicopter echo and its features in time and frequency domains, in a low noise condition.

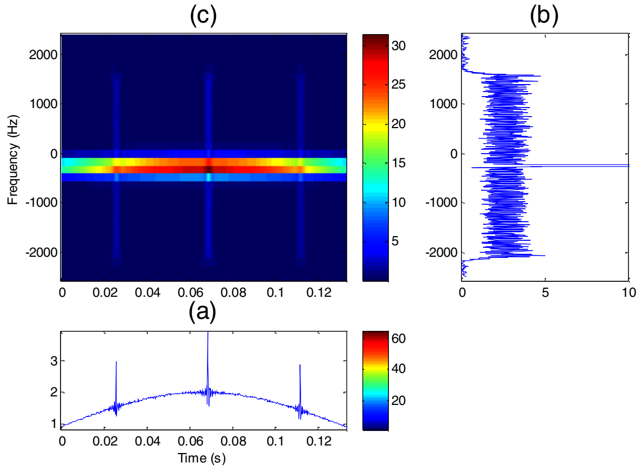

Figure 2 – Helicopter echo with fuselage component: (a) signal in time domain; (b) FFT spectrum of the signal; (c) STFT spectrogram

3. UNDERSAMPLING

To accurately accomplish measuring time between peaks and, specially, blade tip velocity it is necessary for a radar to perform a suitable sampling of the signal, in order to avoid undersampling and aliasing. That is achieved when the sampling rate obeys Nyquist Sampling Theorem [6]. In this case, the sampling frequency must be greater than twice the maximum Doppler frequency possible for an helicopter echo.

Helicopters are designed for having no parts with linear velocity greater than sound velocity, otherwise the noise would be unsupportable for anyone inside or near the helicopter. With this information, we can assure that a radar can avoid undersampling of any helicopter echo if the sampling rate is greater than twice the Doppler frequency due to sound velocity.

In ordinary air defense radars, and in any other ordinary pulse or pulse-Doppler radar, the sampling rate is equal to the pulse repetition frequency (PRF), since each pulse echo generates a sample.

Therefore, the necessary PRF for a radar to avoid undersampling is

$$PRF \geq 2f_{max} = \frac{4v_s f_0}{c} \qquad (5)$$

where f_{max} is the maximum possible Doppler frequency in the helicopter echo, v_s is the sound velocity and f_0 is radar transmission frequency.

The undesired effects of undersampling are represented in Figure 3, where it can be seen, just comparing with Figure 2, that the frequency spectrum is not completely represented, clearly suffering from aliasing. It can also be noticed that, in this case, it is not possible to perform efficient measures in time domain.

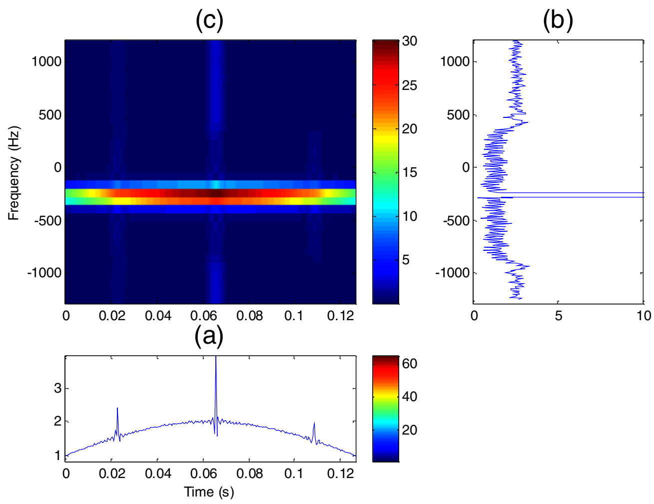

Figure 3 – Undersampled helicopter echo: (a) signal in time domain; (b) FFT spectrum of the signal; (c) STFT spectrogram

4. CLASSICAL TECHNIQUE

The classic and largely used technique for radar based helicopter classification is the L/N quotient technique, presented in [1]. This technique consists in calculating the L/N-quotient, where L is the helicopter blade length and N is the number of blades in the main rotor. This quotient can be calculated by the following relations ([2]):

$$2\pi f_{rot} L = v_{tip} = \frac{f_{tip} c}{2f} \qquad (6)$$

and

$$\begin{cases} f_{rot} N = \frac{1}{\tau}, \text{ if N is even} \\ 2f_{rot} N = \frac{1}{\tau}, \text{ if N is odd} \end{cases} \qquad (7)$$

where f_{rot} is the blade rotation frequency, v_{tip} is the blade tip velocity, f_{tip} is the Doppler frequency due to v_{tip} and τ is time between successive peaks.

Those relations can be combined to allow calculating the L/N-quotient

$$\begin{cases} \frac{L}{N} = \frac{v_{tip}\tau}{2\pi} = \frac{f_{tip}\tau}{4\pi\frac{f}{c}}, \text{ if N is even} \\ \frac{L}{N} = \frac{v_{tip}\tau}{\pi} = \frac{f_{tip}\tau}{2\pi\frac{f}{c}}, \text{ if N is odd} \end{cases} \qquad (8)$$

Therefore, the quotient can be obtained by measuring time between peaks and blade tip Doppler frequency, as shown in Figure 4. Performing these measures can become a difficult task when the signal presents high noise levels or undersampling.

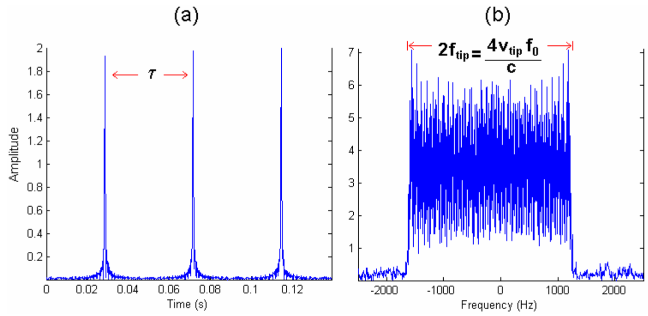

Figure 4 – Obtaining the L/N quotient in the classical model: (a) time between peaks; (b) blade tip velocity

Nowadays, τ and f_{tip} can be directly used as features for a pattern recognition algorithm after a suitable preprocessing.

A more efficient method to measure those features is presented in [2], where the time between peaks is measured by a peak detector over a incoherently integrated signal, while the blade tip Doppler frequency is measured by a sliding windows procedure over a coherently integrated signal.

5. THE CONTINUOUS WAVELET TRANSFORM (CWT)

The Continuous Wavelet Transform of a signal *s(t)* with respect to a wavelet *g(t)* is defined as a matrix of scalar products in $L^2(\mathbb{R})$ ([7]), where which term is given by

$$W(b,a) = \langle s, g_{a,b} \rangle = \int_{t=-\infty}^{+\infty} s(t)\overline{g_{a,b}(t)}dt \qquad (9)$$

where

$$g_{a,b}(t) = \frac{1}{\sqrt{a}} g\left(\frac{t-b}{a}\right) \qquad (10)$$

is a family of wavelets based on the wavelet *g(t)*, called mother wavelet, and generated through a dilation by a factor $a \in \mathbb{R}^*_+$ and a translation by $b \in \mathbb{R}$. Then, the signal can be represented and analyzed in terms of time and scale.

Therefore, the CWT can be written as

$$W(b,a) = \int_{t=-\infty}^{+\infty} s(t) \overline{\frac{1}{\sqrt{a}} g\left(\frac{t-b}{a}\right)} dt \qquad (11)$$

Then, if the mother wavelet is band limited, with central frequency f_c and bandwidth Δf_c, the central frequency of each one of the daughter wavelets, denoted by $g\left(\frac{t-\tau}{a}\right)$, will be $\frac{f_c}{a}$, and the relative bandwidth will be the same for every wavelet in the family.

$$\frac{\Delta f}{f} = \frac{\Delta f_c / a}{f_c / a} = \frac{\Delta f_c}{f_c} = \text{constant} \qquad (12)$$

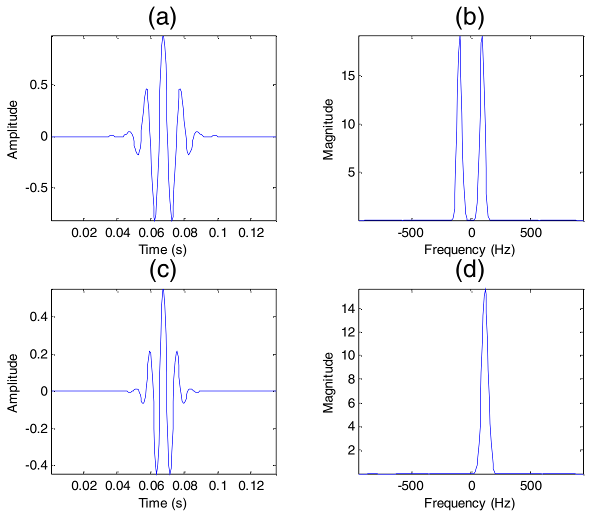

Figure 5 – Morlet Wavelet: (a) real part of the Real Morlet Wavelet in time domain; (b) absolute value of the Real Morlet Wavelet in frequency domain; (c) real part of the Complex Morlet Wavelet in time domain; (d) absolute value of the Complex Morlet Wavelet in frequency domain;

The time domain formulation for CWT can also be analyzed as a correlation between the wavelets family and the signal to be analyzed. This is usually implemented as a convolution between the signal and the complex conjugate of a wavelet copy inverted in time, such as

$$W(b,a)=\int_{t=-\infty}^{+\infty} s(t)\frac{1}{\sqrt{a}}g^{-}\left(\frac{b-t}{a}\right)dt = s\otimes g^{-}_{a,b} \tag{13}$$

where

$$g^{-}_{a,b}(t)=\overline{g_{a,b}(-t)} \tag{14}$$

This kind of formulation facilitates foreseeing the result of the CWT of a real signal and a real wavelet. However, it is difficult to analyze the convolution results when there is a complex signal involved. Therefore, it is very interesting to look for a formulation of the CWT in frequency domain.

5.1. CWT in Frequency Domain

Starting from equation (10), one can apply the Fourier transform, obtaining

$$G_{a,b}(\omega)=\sqrt{a}G(a\omega)e^{-j\omega b} \tag{15}$$

Using Parseval's Theorem, it can be derived a frequency domain formulation for CWT ([8])

$$W(b,a)=\sqrt{a}\int_{\omega=-\infty}^{+\infty} S(\omega)\overline{G(a\omega)}e^{j\omega b}d\omega \tag{16}$$

5.2. Complex Signals Analysis

Using CWT to analyze complex signals, some details have to be taken into account. When analyzing a complex signal using CWT with a real wavelet, the positive and negative parts of the frequency spectrum are overlapped, since the real wavelet has redundant spectrum for both positive and negative frequencies.

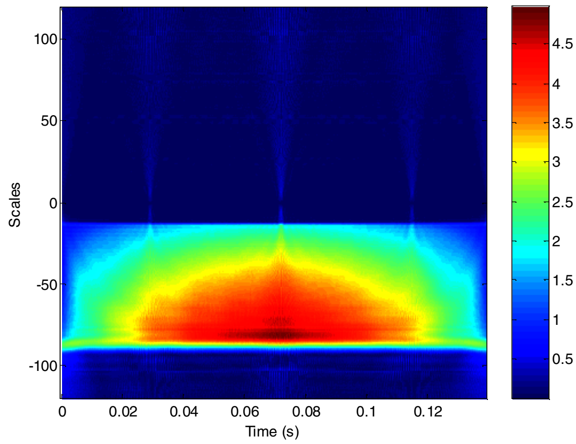

Figure 6 – Adapted Continuous Wavelet Transform

Otherwise, when using a complex wavelet, that has only positive frequency components, to analyze a complex signal, only positive frequency components of the signal are analyzed, while negative frequency components are ignored.

Figure 5 illustrates the behaviour of real and complex wavelets, where it can be noticed the spectral redundancy of the real wavelet and the positive only spectrum of the complex wavelet.

A solution to completely analyze complex signals is to perform an adapted wavelet transform, consisting in calculating two CWT's, being one with the standard wavelet and other with a copy of the first wavelet inverted in frequency, and concatenating the results to have the same kind of analysis for both negative and positive frequencies.

Figure 6 shows an adapted CWT, where the upper half of the figure, indicated by positive scales, represents the regular CWT of the signal, which analyzes positive spectrum components. The lower half of the figure, indicated by negative scales, is the CWT of the signal relative to the frequency inverted copy of the wavelet, analyzing negative components of the signal.

The signal used in this example is an echo from a helicopter with negative Doppler frequency; therefore, it would not be suitably analyzed by a regular CWT with complex wavelet.

6. PROPOSED TECHNIQUE

6.1. Using CWT to Measure Time Between Peaks

A good feature of the CWT is its capability of detecting discontinuities, since it decomposes signals into elementary building blocks that are well localized both in space and frequency [9]. This feature is used here to find the peaks due to the helicopter main rotor blades.

Several wavelets were tested with the helicopter echo and the best result was achieved for the complex Shannon wavelet

$$g(x)=\sqrt{f_b}\,\mathrm{sinc}(f_b x)e^{j2\pi f_c x} \tag{17}$$

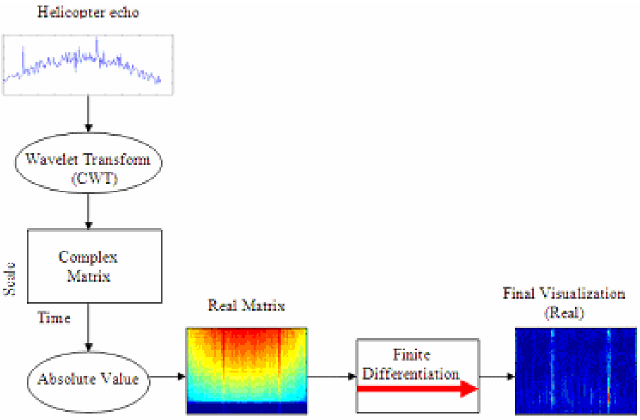

Figure 7 – Finite differentiation of the CWT

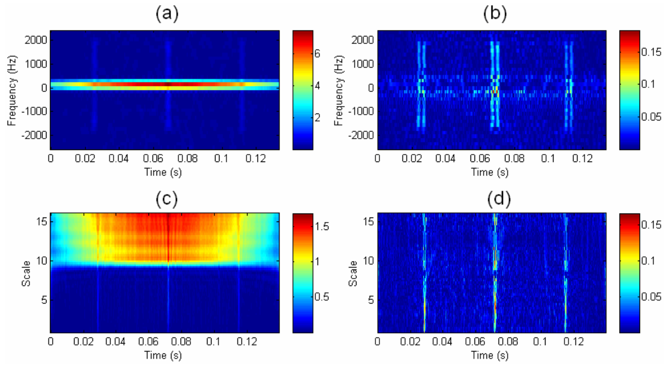

Figure 8 – Effect of finite differentiation in STFT and CWT: (a) STFT spectrogram of the signal; (b) finite differentiation of the STFT; (c) CWT of the signal; (d) finite differentiation of the CWT

where f_c is the central frequency and f_b is a bandwidth parameter ($\Delta f_c \sim f_b$).

In frequency domain, the complex Shannon wavelet is a rectangular pulse centered in f_c and with width proportional to the parameter f_b.

$$G(\omega)=\begin{cases}\dfrac{a}{\sqrt{f_b}}, \text{ if } -\dfrac{a\pi}{f_b}+f_c \le \omega \le \dfrac{a\pi}{f_b}+f_c \\ 0, \text{ otherwise}\end{cases} \qquad (18)$$

6.2. Using Finite Differences to Eliminate Fuselage Component

When using CWT over a helicopter echo signal, it appears a strong component due to the fuselage echo, masking the blades echo and turning it hard to measure the time between successive peaks. This phenomenon occurs not only for CWT, but with any kind of joint time-frequency analysis.

The strong component can be eliminated by differentiating the CWT of the helicopter echo in time direction. It makes transitions in time to stand out, while transitions in scale (frequency) are ignored.

$$W_{diff}(b, a) = W(b, a) - W(b-1, a) \qquad (19)$$

Figure 7 illustrates the process workflow, from the original helicopter echo through the final visualization after the differentiation of the CWT. This technique of applying a finite differentiation to eliminate fuselage echoes can also be used with other joint time-frequency techniques.

As illustrated in Figure 8, this method was also tested with STFT spectrogram. It can be noticed that it is possible to use finite differentiation along with STFT spectrogram to make it easier to measure blade tip Doppler frequency in signals with higher noise levels.

However, the best results were achieved using CWT, and the time between peaks have been measured with the samples of a single dwell even when the signal is undersampled.

6.3. Teager-Kaiser energy

Usually, the energy of a signal is calculated by taking the square of the instantaneous amplitudes. However this method does not take into account the signal variation in time.

It is shown in [10] that instantaneous energy of a discrete time signal can be estimated using the amplitude samples and their neighbors in time.

$$E \bullet x[n]x^*[n] - x[n+1]x^*[n-1] \qquad (20)$$

The original Teager-Kaiser energy when it is expressed by Equation (20) is not effective when applied to signals which have multi frequency components. Then, as suggested by [11], the Teager-Kaiser energy can be applied along each scale after calculating the CWT.

When the Teager-Kaiser energy had been applied to the CWT finite differentiation, it turned out to be very useful in noise reduction, since this energy makes the signal features to stand out against the noisy background, as it is illustrated in Figure 9.

6.4. Wavelet transform modulus maxima (WTMM)

In Figure 9 it can be seen that blade peaks stand out after CWT and finite differentiation in time. The next step is, therefore, to find a way of measuring the position in time of these peaks, in order to make it possible to calculate time between successive peaks.

An efficient method to measure peaks positions in CWT can be found in [12], where it is said that signal transitions and singularities can be found by the first derivative of the CWT.

This method is called Wavelet Transform Modulus Maxima (WTMM). It consists in finding the local maxima of the CWT and linking them to produce lines called Wavelet Transform Modulus Maxima Lines (WTMML). The WTMML are illustrated in Figure 10.

With the localization of these lines in time direction, it is possible to calculate the time between successive helicopter echo peaks.

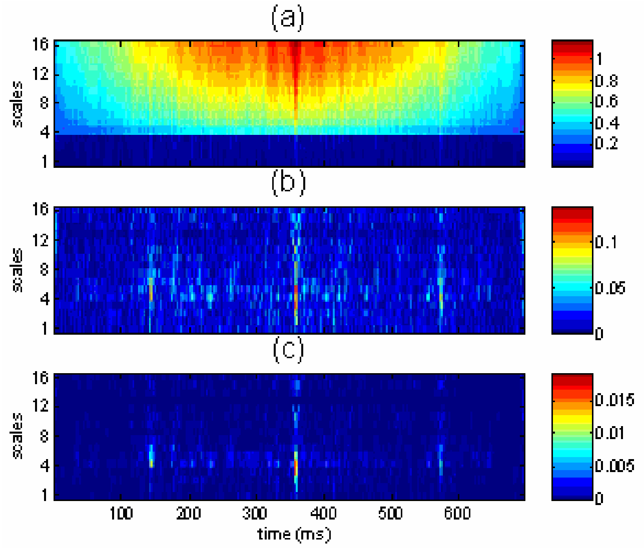

Figure 9 – (a) CWT of the signal; (b) CWT of the signal after finite differentiation; (c) Teager-Kaiser energy of the CWT after finite differentiation

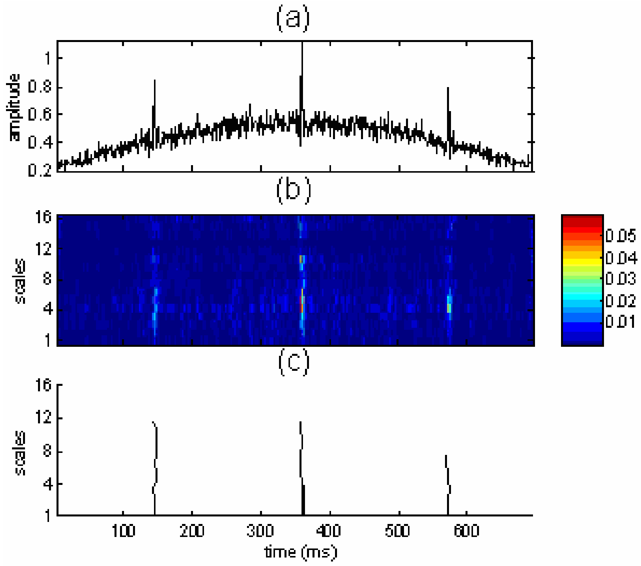

Figure 10 – (a) Signal in time domain; (b) Teager-Kaiser energy of the CWT after finite differentiation; (c) Wavelet transform modulus maxima lines (WTMML).

CONCLUSIONS

Radar based helicopter classification is not an easy task specially under noisy environment, and this paper shows that Continuous Wavelet Transform can be a very useful tool when trying to determine one of the most important features for classifying helicopters, the time between successive peaks.

In [2], a way of finding the blade tip velocity from a coherently integrated radar signal is presented. This paper presents a way of measuring time between peaks also using a coherently integrated radar signal.

A conclusion achieved from this work is the expression for the minimum radar pulse repetition frequency (PRF) necessary for avoiding aliasing in the helicopter echo presented in equation (5).

It is important to mention that in the case where the radar PRF does not obey the equation (5) and the aliasing occurs, it is almost impossible to measure blade tip velocity and, therefore, in this case the time between peaks can be the only feature available when trying to perform helicopter classification. In these cases, the method presented in this paper for measuring time between peaks also achieved good results.

Finally, the use of other techniques, as the Wavelet transform modulus maxima and the Teager-Kaiser energy, shows that many other tools already existent in signal processing can turn out to be very useful in the determination of features for this task.

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