Brazilian deep water carbonate reservoir study using the Wavelet Transform Teager-Kaiser Energy
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Summary
Spectral decomposition has proven to be a powerful means to identify strong amplitude anomalies at specific frequencies that are otherwise buried in broad-band response. We compute Teager-Kaiser Energy for each component of a joint time-frequency representation to generated from a 3D survey acquired over a Brazilian deep water carbonate reservoir. This nonlinear energy tracking algorithm allows us to differentiate between high amplitude reservoir and other high amplitude reflections. We calibrate our algorithm against synthetic seismic traces generated from the well logs and and then apply to the real seismic data to reveal important geological features.

Introduction
Usually, we are not concerned with the total energy of a wave but rather with the energy in the vicinity of the point we observe it (Sheriff and Geldart, 1995). The most common means to estimate the energy of any kind of signal is by simply squaring their amplitude samples. A slightly more sophisticated means is to compute the envelope of the complex trace. Kaiser (1990) introduced a much more local energy estimate valid for monofrequency signals that takes into account the neighbourhood of each time sample.

Matos and Johann (2007) applied Kaiser’s (1990) idea to seismic signals through the use of the continuous wavelet transform and showed that the Teager-Kaiser energy is directly associated with the energy of the seismic wavefield. They applied this workflow to a clastic reservoir and used the resulting time-frequency representation to detect and track anomalously strong seismic events that were otherwise buried in the broad spectral response.

In this paper we apply the same workflow to seismic data acquired over a Brazilian carbonate reservoir seismic data.

The Wavelet Transform Teager-Kaiser Energy
It is well known that the seismic energy density, \( E \), for a monofrequency signal can be expressed as
\[
E = \frac{1}{2} \rho \omega A^2 = 2\pi^2 \rho f A^2
\]
and is proportional to the density of the medium, \( \rho \), to the square of the frequency, \( \omega \) (in radians) or \( f \) (in Hz), and amplitude, \( A \), of the wave (Sheriff and Geldart, 1995, p. 58).

Using an analogue mass-spring physical model, Kaiser (1990) proved that the energy of a discrete time signal at time \( t = n\Delta t \) can be expressed as:
\[
E(n) = \frac{1}{2} m \omega^2 A^2 (n) \equiv x^2(n) - x(n+1)x(n-1)
\]  
(2)
where \( m \) is the mass of the object suspended by a spring, \( A \) is the amplitude of the oscillation and \( x(n) \) are the samples of the discrete time signal. If we consider the discrete mass, \( m \), in equation 2 to be a lumped approximation of the continuous density, \( \rho \) in equation 1, we note that these two equations are identical such that we can use equation 2 to estimate the instantaneous Teager-Kaiser (TK) energy of the seismic wavefield. Equation (2) is strictly true for a mono-frequency wavefield. Interpreting the wavelet transform as a joint time-frequency representation of the seismic wavefield that can be implemented through band-pass filter banks (Mallat, 1999). Matos and Johann (2007) estimated the joint time-frequency distribution using the continuous wavelet transform (CWT) and computed the TK energy for each scale (or band pass filter).

Figure 1 illustrates how our wavelet transform Teager-Kaiser energy (WaveTeKE) algorithm works. Figure 1a shows the acoustic impedance measured in an offshore Brazilian well through a producing carbonate reservoir. Figure 1b is the corresponding synthetic seismogram, Figure 1c is the time-frequency amplitude, \( A(f,t) \) and Figure 1d is the time-frequency TK energy, \( E(f,t) \). Note the strong event at \( t=1.9 \) s in Figure 1e that corresponds to the carbonate reservoir oil field.

Application
Masaferro et al., (2004) state that the combined effects of variation in depositional facies and diageneric alteration play a key role in controlling variations in sonic velocities and thus is acoustic impedance in carbonates. Chopra and Marfurt (2007) show how the shape or geomorphology of reflection patterns, coupled with appropriate models deposition and diagensis, further aid the mapping of carbonate facies. Thus, both geometric (that measure lateral changes) and trace shape (that measure the vertical seismic waveform) seismic attributes can be a grade aid in the characterization of carbonate reservoirs. Figure 2b-d shows horizon slices through several popular geometric attribute volumes along the top of the horizon displayed in Figure 2a. The two white circles represent producer wells while the two white crosses represent non-producer wells. Figure 3a shows a horizon slice along the...
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Figure 1: The WaveTeKE algorithm applied to a Brazilian carbonate well log: (a) Acoustic impedance; (b) Synthetic seismic trace; (c) The wavelet transform scalogram obtained squaring the real-valued Morlet wavelet CWT coefficients; (d) The wavelet transform displayed as a filter bank followed by the Teager-Kaiser Energy estimator; and (e) The WaveTeKE result indicates an energy peak at $t = 1.87$ s and $f = 28$ Hz.

The seismic trace spectrum is directly related to the shape of the seismic trace. Pearson and Hart (2004) showed that spectral components can be used in carbonate reservoir characterization. Specifically, they predicted the porosity of a carbonate reservoir from a linear combination of the slope from peak to maximum spectral frequency and the ratio of the number of positive samples over the number of negative samples within a time interval.

Previous work by Matos and Osório (2005), Matos et al. (2005), and Liu and Marfurt (2007) showed how the maximum (or ‘peak’) spectral frequency and its associated amplitude can be related to important geologic features. Here, we will apply the WaveTeKE measurement to time-frequency attributes and show how they are directly correlated to trace shape.

Following Matos et al. (2005), we extract the peak energy using the WaveTeKE algorithm given by equation 2 for the whole 3D volume. Figures 4b and 4c display the peak energy and peak frequency, while Figure 4a illustrates a formation attribute obtained by picking the maximum amplitude within an 80 ms time interval attribute below the reservoir. We note that the reservoir area is delineated. As a reference, in both Figures 4b and 4c the upper yellow picks represent the top of the reservoir while the vertical black dashed lines represent the wells. These lines confirm the value of the proposed algorithm, where producers are correlated to high, high frequency reflectors while non producers are related to small energy reflections.

Figure 5a shows the 80 ms time interval attribute that displays the minimum amplitude (strongest negative value) attribute below the reservoir top that is well-established in this field to be a good indicator of reservoir quality. Figure 5b shows the time interval maximum WaveTeKE frequency and its associated amplitude plotted together using a 2D colour bar.

Conclusions
We show that the Teager-Kaiser energy can be computed for seismic data through the joint time-frequency representation. The TK energy appears to be quite effective in delineating strong amplitude, high frequency events associated with a producing areas of a carbonate reservoir. The results obtained with real seismic data show the WaveTeKE potential use as an exploratory tool to detect energy associated with important geological marks and potential exploratory leads.

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Figure 2: (a) Time-structure map of the the top of the reservoir. Horizon slices along the top horizon map through (b) most-positive curvature, (c) semblance coherence, and (d) inline amplitude gradient. Notice the strong inline and crossline acquisition footprint in the curvature map.

Figure 3: (a) Horizon slice along the top reservoir and (b) vertical slices AA′, and (c) BB′ through the seismic amplitude volume. Upper black pick denotes the top of the reservoir.
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Figure 4: (a) Horizon slice through the TE energy volume. (b) Formation attribute map of the TE absolute sum value between the top Horizon and 80 ms below WaveTeKE instantaneous amplitude along (c) line AA’ and (d) line BB’.

Figure 5: 80-ms time interval seismic attributes: (a) Minimum amplitude and (b) $\max_{f,t} |E(f,t)|$ given by equation 2. White circles represent producer well locations. Crosses represent non-producer well locations.
EDITED REFERENCES
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REFERENCES