

Continuous wavelet transform phase residues applied to detect stratigraphic discontinuities

Marcílio Castro de Matos*, Instituto Militar de Engenharia, Brasil, Oswaldo Davogusto, Kui Zhang and Kurt J. Marfurt, The University of Oklahoma

Summary

Spectral decomposition has proven to be a powerful means of identifying strong amplitude anomalies at specific frequencies that are otherwise buried in the broad-band response. To date, most publications have focused on using the spectral magnitude components to identify lateral changes in stratigraphy, wavefield attenuation due to Q , and unconformities between geologic formations. Although seismic acquisition and processing does a much better job in preserving phase, little has been published about interpreting the phase components resulting from spectral decomposition. In this paper, we show how Morlet complex wavelet transform (CWT) phase residues can be used to improve seismic spectral decomposition interpretation by detecting the phase discontinuities in the joint time-frequency spectral phase component. Using synthetic models we illustrate how phase singularities can be associated to geologic features and at the end we illustrate how phase residues help the interpretation of the Anadarko Basin Red Fork channels of Oklahoma, USA.

Introduction

Seismic interpretation is based on amplitude variation with time and space. Usually, interpreters look for relatively coherent seismic amplitudes that represent impedance contrasts associated with geologic boundaries (Henry, 2004). Seismic amplitudes can be modeled using a simple convolutional operation between the reflection coefficient series that represents the geology and the seismic wavelet that represents an impulsive seismic source after passing through the earth filter.

Stratigraphic variations due to changes in the depositional systems or diagenesis (in carbonates) (Hart, 2008) generate different reflectivity coefficient series for each environment and, consequently, different kinds of seismic waveforms. In addition to phase changes due to stratigraphy, phase rotations can be introduced through seismic processing and may be further adjusted by the interpreter (Roden and Sepulveda, 1999). Some of the geology-induced phase shifts can be easily identified, such as spatial discontinuities associated with faults and incised channels, but phase shifts due to condensed sections and erosional unconformities can be quite subtle.

In this paper, we use the Morlet complex continuous wavelet transform (CWT) spectral decomposition to measure phase changes as a function of the frequency, and show how such changes can be related to seismic stratigraphy. We begin with a simple review of spectral decomposition, with an emphasis on the phase rather than

the magnitude component. Then instead of attempting to unwrap the phase (a fairly difficult problem), we show how we can simply compute a phase residue, or anomalies in the phase spectra. Finally, we calibrate these phase residues computed from a seismic data volume acquired over the Anadarko Basin, Oklahoma, USA, against well control to validate the resolution of thin sands that fall at the limit of seismic resolution.

Spectral decomposition – generating time frequency amplitude and phase spectra

Partyka et al. (1999) first showed how the seismic spectral response from a short time window can be used to map lateral changes in acoustic properties and thickness of stratigraphic layers. Since then, spectral decomposition has become a widely-accepted interpretation tool and has been applied to reservoir characterization (Matos et al., 2005, Liu and Marfurt, 2007), hydrocarbon detection (Castagna et al., 2003) and stratigraphic analysis (Hall and Trouillot, 2004). Most spectral decomposition applications are based on the magnitude component of the joint time-frequency spectrum. Although some of these techniques are quadratic-energy based, such as the Wigner-Ville distribution, and do not generate phase information, the short-window discrete Fourier transform, wavelet transform, and matching pursuit algorithms all produce complex spectra that can be represented by magnitude and phase components. Phase spectra respond to lateral discontinuities and has been successfully used to delineate faults. However, to our knowledge, very little has been published on the spectral phase response due to stratigraphy, or on mapping spectral phase discontinuities.

Joint time-frequency phase unwrapping

Spectral decomposition of a seismic trace generates complex data that can be decomposed into magnitude and phase as a function of time and frequency. For each frequency, f , the phase $\phi(t,f)$, is a measure of travel time distance from some reference time, t_0 . One might anticipate that the time-frequency phase unwrapping can be simply solved by unwrapping each phase of each frequency component using the one-dimensional technique (Leonard, 2007). However, unwrapping the phase using this “component independent” process will generate phase discontinuities between adjacent frequency components. This shortcoming suggests accounting for the phase relationships along the frequency as well as the time axis to unwrap the data, thereby requiring a two-dimensional phase unwrapping methodology.

CWT phase residues applied to detect stratigraphic discontinuities

Basically, two-dimensional phase unwrapping is embodied in the solution of the line integral:

$$\varphi(r) = \int_{\Gamma} \nabla \varphi dr + \varphi(r_0) \quad (1)$$

which is Itoh's (1982) equation expressed in two or more dimensions. Aliasing, singularities, and noise can make equation 1 highly dependent on the integration path. For this reason, it is very important to know which pitfalls should be avoided in defining the path Γ in equation 1.

Ghiglia and Pritt (1998) provide an excellent survey of 2D phase unwrapping techniques and show how a complex residue theorem based on vector calculus can be applied to the phase unwrapping problem. Specifically, they chose the smallest possible path defined by 2 time sample by 2 frequency component rectangular window for every point in the wrapped $\varphi(t, f)$, giving

$$I = \frac{W\{\psi(t + \Delta t, f) - \psi(t, f)\}}{2\pi} + \frac{W\{\psi(t + \Delta t, f + \Delta f) - \psi(t + \Delta t, f)\}}{2\pi} + \frac{W\{\psi(t, f + \Delta f) - \psi(t + \Delta t, f + \Delta f)\}}{2\pi} + \frac{W\{\psi(t, f) - \psi(t, f + \Delta f)\}}{2\pi} \quad (2)$$

where W is a wrapping operator that wraps its argument value into the range $(-\pi, +\pi]$ by adding or subtracting an integer multiple of 2π radians to its argument.

If I in equation 2 is non-zero there are inconsistent points which Ghiglia and Pritt (1998) call phase residues. Figure 1 shows how the residue is calculated for a small portion of a typical wrapped time-frequency phase matrix.

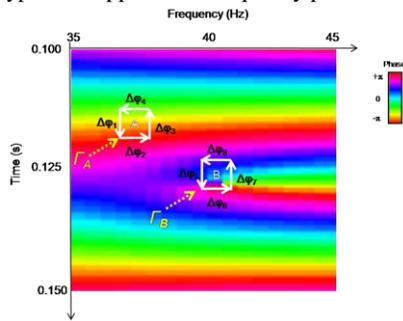


Figure 1. Computation of the phase residue, I , as defined by Ghiglia and Pritt (1998).

Bone (1991) proved that the only possible values for the phase residue are 0 and ± 1 . Workers who choose to unwrap the phase try to avoid the phase residue in some manner. Our objective is much less ambitious. Instead of unwrapping the phase, we will simply display phase residuals properties that appear in the joint time-frequency distribution as seismic attributes that can be associated with stratigraphic discontinuities and busts in seismic data quality.

In this paper we used the Morlet complex CWT as a time-frequency spectral decomposition that can be defined as the

convolution between the seismic trace and the time-reversed complex Morlet wavelet. Before we introduce the proposed seismic attributes we review some CWT phase properties using three simple examples. Figure 2a shows a spike signal and Figure 2b and 2c illustrates its CWT magnitude and phase respectively. Since the reflectivity is a simple delta function, the CWT is a reproduction of the mother wavelet at the time location of the delta function for each of the different scales. Consequently, we can clearly detect the spike by mapping the ridge of the CWT magnitude or by noting that the lines of constant CWT phase converge to the singularity point (Holschneider, 1995, p. 45). It can also be shown that discontinuities detected from the CWT unwrapped phase are associated with signal singularities (Matos et al., 2009).

When we add a second spike to the signal as showed in Figure 3a, simulating a low impedance reflectivity time series, we observe a similar phase and magnitude pattern at the higher frequencies. However, at the lower frequencies, residues appear in the CWT phase. Note that the residues appear between, and not at the spikes (Figure 3c). Note also that the CWT magnitude is very low at the residue location compared to the magnitude near the spike locations. We will use this magnitude later to weight whether residue is significant or not.

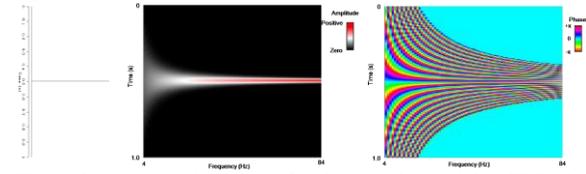


Figure 2. a) A single-spike reflectivity series and its CWT (b) magnitude and (c) phase spectra.

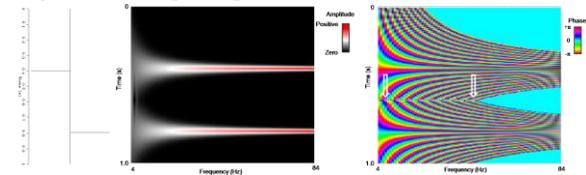


Figure 3. a) A double reflectivity series and its CWT (b) magnitude and (c) phase spectra. White arrows indicate phase residues.

Seismic data can be modeled by convolving the reflectivity with a seismic wavelet producing a band-pass filtered version of the reflectivity time series. Therefore, the CWT of a seismic trace can be interpreted as a cross correlation between the scaled Morlet wavelets and the seismic wavelet located at each reflectivity point where there is impedance contrast. Figure 4a shows the CWT magnitude and phase of a seismic trace with thickness close to zero taken from the same wedge model shown in Figure 4e. Compared with Figure 3, the magnitude is smeared at high frequencies but the constant phase lines still converge to the location of the spike.

CWT phase residues applied to detect stratigraphic discontinuities

Extending the CWT analysis to representative traces of the same wedge model, Figure 4 shows the CWT amplitude and phase of representative traces and their corresponding phase residues. Contrary to the spike signal, now we can note that the residues correlate to subtle phase changes in the joint time-frequency distribution.

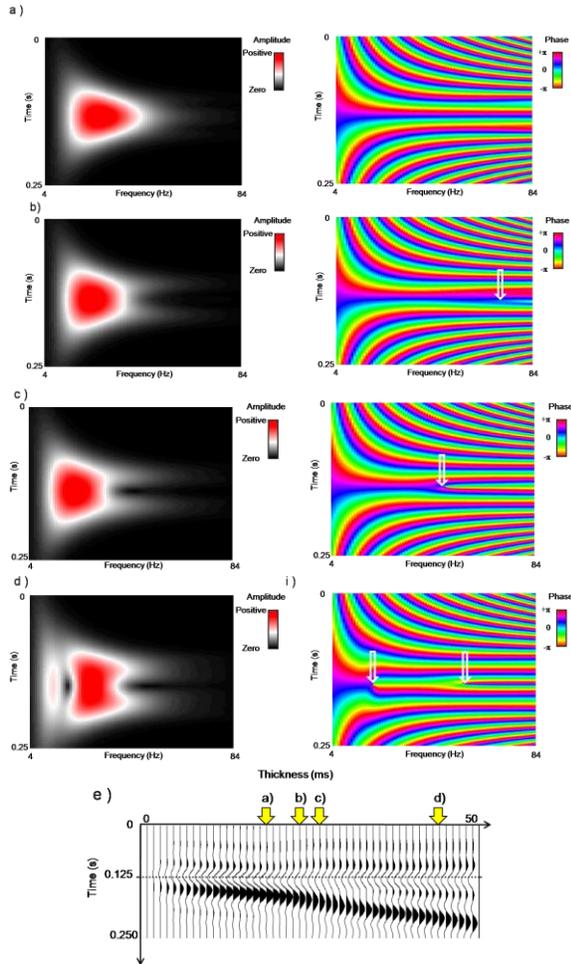


Figure 4. (a)-(d) CWT amplitude and phase of the four seismic traces shown in (e). White arrows indicate phase residues.

Based on these examples, we defined three new seismic attributes: the frequency where each phase residue occurs, the phase value at the residue location and the corresponding magnitude at this frequency, f , and time, t .

We test these attributes on a synthetic channel seismic model designed by mirroring the Figure 4e wedge model (Figure 5a). By plotting the maximum amplitude at the phase residues in Figure 5b we can clearly see the detected channel. As expected, the proposed attributes did not detect the main reflectors and are time, or depth, shifted from them. Actually, as shown in Figure 5c they can be

associated with subtle stratigraphic interference phenomena which we will discuss in greater detail with real data example.

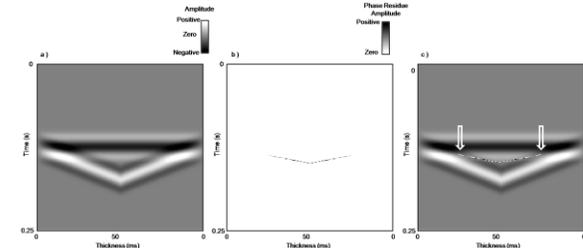


Figure 5. (a) Channel model generated by reflecting the wedge model shown in Figure 4e about the origin. (b) Phase residue modulated by the magnitude shows the channel interference pattern. (c) Figures (a) and (b) overlaid.

Application

To demonstrate the value of phase residues we use a seismic data volume that served as one of the first published applications of spectral decomposition (Peyton et al., 1998). Ten years and 100s of wells later in this survey, the Red Fork channels of the Anadarko Basin of Oklahoma are still problematic. The incised valleys described by Peyton et al. (1998) have undergone at least five stages of incision and fill. Suarez et al. (2008) report that the fill can be comprised of lag deposits, shales, coals, muddy sands, and sands. In addition to the seismically-resolved incised channels, Suarez et al. (2008) report the common occurrence of ‘invisible channels’ – channels that are seen by the drill bit and well logs but not by seismic amplitude data. While the top and bottom of the Redfork interval can be resolved seismically, the internal reservoir facies often fall below seismic resolution.

We have applied the Morlet continuous wavelet transform (CWT) to the seismic data volume, computed the phase residue seismic attributes proposed in this paper, and overlaid the results against gamma-ray (GR) well log response allowing us to correlate phase residue anomalies with vertical changes in lithology. Figure 6 shows the location for the composite seismic lines and the locations of the 15 wells used in the interpretation.

Examining composite line A-A’ (Figure 7) where we co-render the seismic amplitude, the phase residues and gamma-ray logs, we recognize a correlation of the sand signatures in the GR logs with lineaments in the phase residue. The phase residue amplitude and frequency were blended using a two-dimensional colormap (Guo et al., 2008) and allow us to interpolate these thin sands between the sparse well control. These sands are not apparent in the seismic amplitude data due to the limited vertical resolution. Figures 8 displays one of the vertical sections crossing well Jay Jay 2 with key features defined by the phase residue attribute. Peyton et al. (1998) and Suarez et

CWT phase residues applied to detect stratigraphic discontinuities

al. (2007) gave special importance to this well that is producing from a stage III channel sand of 50 ft, but was not imaged due to the moderate seismic data quality.

Conclusions

We show how phase residues introduced in signal processing by Ghiglia and Pritt (1998) can be applied to Morlet CWT time-frequency distributions generated by spectral decomposition. CWT phase residues are related to transitions between different phase values and although they are not a thin bed detection tool, they do reveal important stratigraphic features. Through calibration using 15 wells and a 350 km² seismic survey we interpret these phase residues appear to be sensitive to subtle discontinuities that are not easily seen in input seismic amplitude data. We believe phase attributes are sensitive to the same kinds of stratigraphic discontinuities seen by analysis of the magnitude component of time-frequency distribution described by Matos et al. (2007) using wavelet transforms and Li and Liner (2008) using the continuous wavelet transform. Since the phase is often a more robust seismic measure than magnitude (Ulrych, 2008), we believe it holds significant promise in mapping stratigraphic unconformities.

Acknowledgments

The authors would like to thank Chesapeake for providing the data used in this study and the permission to publish this work as well as the industry sponsors of the University of Oklahoma Attribute-Assisted Seismic Processing and Interpretation (AASPI) Consortium.

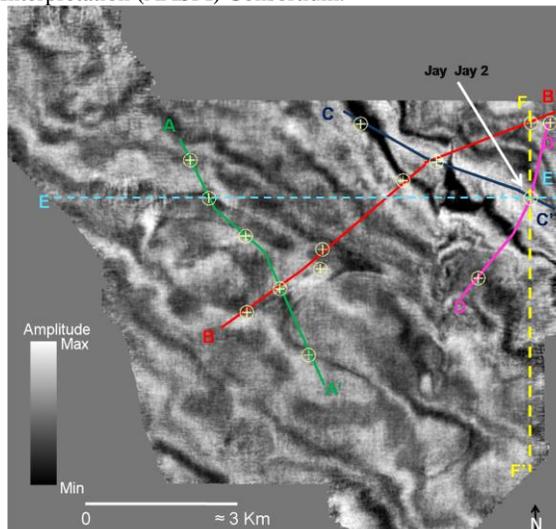


Figure 6. Phantom horizon slice 80 ms below the Skinner through the seismic amplitude volume, indicating lines A-A', B-B', C-C', D-D', E-E' and F-F' through key wells.

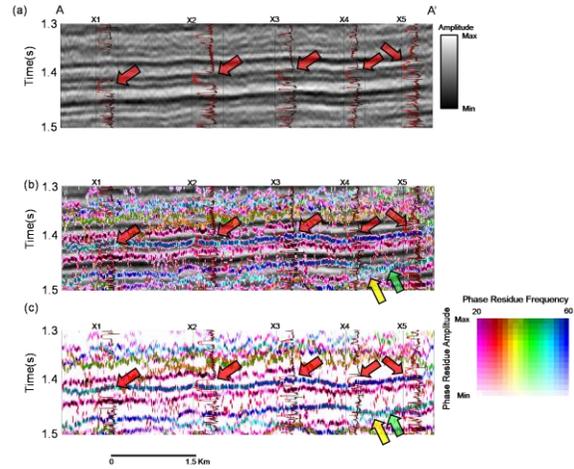


Figure 7. Vertical section A-A' through (a) seismic amplitude data, (b) seismic amplitude data co-rendered with CWT phase residue attributes using two dimensional colormap (Guo et al., 2008) and (c) CWT phase residue attributes using two dimensional colormap (Guo et al., 2008). Red arrows indicate the base of a GR sequence. Yellow arrow indicates a channel-like feature in the CWT phase residue attributes that is masked in the seismic data. The green arrow is interpreted as the levee of the channel interpreted by the yellow arrow or an adjacent channel that is not resolved in the seismic section. In (c) the sand body reflection interface is better resolved in the phase residue attribute; notice that the channel-like features present (green and yellow arrows).

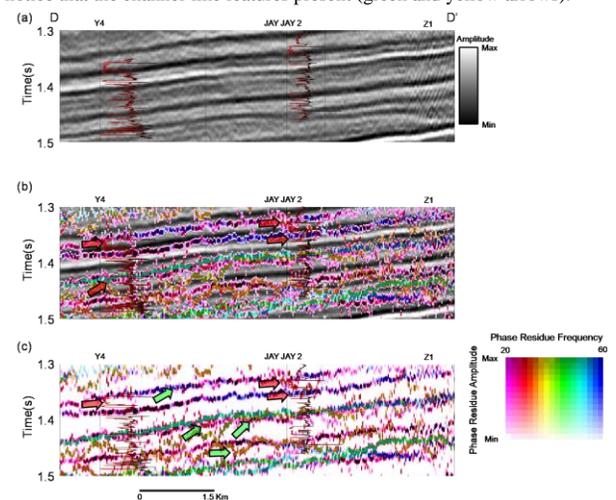


Figure 8. (a) Vertical section D-D' through the seismic amplitude data, (b) seismic amplitude data co-rendered with CWT phase residue attributes using two dimensional colormap (Guo et al., 2008) and (c) CWT phase residue attributes using two dimensional colormap (Guo et al., 2008). In this section no distinctive features are imaged in the seismic data, this is probably due to seismic resolution. Peyton et al. (1998) referred to well Jay Jay 2 in their 1998 work, and stated that this well produces from approximately 50 ft of sand and that no channels were imaged in the area near the well. Channel-like features are found at different stratigraphic levels indicated by the red arrows in wells Jay Jay 2 and Y4. (c) Some other channel like features are identified in the phase residue section (green arrows).

EDITED REFERENCES

Note: This reference list is a copy-edited version of the reference list submitted by the author. Reference lists for the 2010 SEG Technical Program Expanded Abstracts have been copy edited so that references provided with the online metadata for each paper will achieve a high degree of linking to cited sources that appear on the Web.

REFERENCES

- Bone, J. B., 1991, Fourier fringe analysis: the two-dimensional phase unwrapping problem: *Applied Optics*, **30**, no. 25, 3627–3632, [doi:10.1364/AO.30.003627](https://doi.org/10.1364/AO.30.003627).
- Castagna, J., S. Sun, and R. Siegfried, 2003, Instantaneous spectral analysis: Detection of low-frequency shadows associated with hydrocarbons: *The Leading Edge*, **22**, no. 2, 120–127, [doi:10.1190/1.1559038](https://doi.org/10.1190/1.1559038).
- Ghiglia, D., and M. D. Pritt, 1998, *Two-Dimensional Phase Unwrapping: Theory, Algorithms, and Software*: Wiley-Interscience.
- Guo, H., S. Lewis, and K. J. Marfurt, 2008, Mapping multiple attributes to three- and four-component color models - A tutorial: *Geophysics*, **73**, no. 3, W7–W19, [doi:10.1190/1.2903819](https://doi.org/10.1190/1.2903819).
- Hall, M., and E. Trouillot, 2004, Predicting stratigraphy with spectral decomposition: 2004 National Convention, CSEG, Expanded Abstracts.
- Hart, B., 2008, Stratigraphically significant attributes: *The Leading Edge*, **27**, no. 3, 320–324, [doi:10.1190/1.2896621](https://doi.org/10.1190/1.2896621).
- Henry, S., 2004, Understanding seismic attributes:
<http://www.searchanddiscovery.net/documents/2004/henry/images/henry.pdf>, accessed August 10, 2009.
- Holshneider, M., 1995, *Wavelet an analysis tool*: Oxford University Press.
- Itoh, K., 1982, Analysis of the phase unwrapping algorithm: *Applied Optics*, **21**, no. 14, 2470, [doi:10.1364/AO.21.002470](https://doi.org/10.1364/AO.21.002470). [PubMed](#)
- Léonard, F., 2007, Phase spectrogram and frequency spectrogram as new diagnostic tools: *Mechanical Systems and Signal Processing*, **21**, no. 1, 125–137, [doi:10.1016/j.ymssp.2005.08.011](https://doi.org/10.1016/j.ymssp.2005.08.011).
- Li, C., and C. Liner, 2008, Wavelet-based detection of singularities in acoustic impedances from surface seismic reflection data: *Geophysics*, **73**, no. 1, V1–V9, [doi:10.1190/1.2795396](https://doi.org/10.1190/1.2795396).
- Liu, J., and K. Marfurt, 2007, Instantaneous spectral attributes to detect channels: *Geophysics*, **72**, no. 2, P23–P31, [doi:10.1190/1.2428268](https://doi.org/10.1190/1.2428268).
- Matos, M. C., K. Zhang, K. J. Marfurt, and R. Slatt, 2009, Stratigraphic discontinuities mapped through joint time-frequency seismic phase unwrapping: 79th Annual International Meeting, SEG, Expanded Abstracts, 28, 1087-1091.
- Matos, M. C., P. L. M. Osorio, E. C. Mundim, and M. A. S. Moraes, 2005, Characterization of thin beds through joint time-frequency analysis applied to a turbidite reservoir in Campos Basin, Brazil: 75th Annual International Meeting, SEG, Expanded Abstracts, 24, 1429-1432.
- Partyka, G., J. Gridley, and J. Lopez, 1999, Interpretational applications of spectral decomposition in reservoir characterization: *The Leading Edge*, **18**, no. 3, 353–360, [doi:10.1190/1.1438295](https://doi.org/10.1190/1.1438295).

- Peyton, L., R. Bottier, and G. Partyka, 1998, Interpretation of incised valleys using new 3-D seismic techniques: A case history using spectral decomposition and coherency: *The Leading Edge*, **17**, no. 9, 1294–1298, [doi:10.1190/1.1438127](https://doi.org/10.1190/1.1438127).
- Roden, R., and H. Sepulveda, 1999, The significance of phase to the interpreter: practical guidelines for phase analysis: *The Leading Edge*, **18**, no. 7, 774–777, [doi:10.1190/1.1438375](https://doi.org/10.1190/1.1438375).
- Suarez, Y., K. J. Marfurt, and M. Falk, 2008, Seismic attribute-assisted interpretation of channel geometries and infill lithology: A case study of Anadarko Basin Red Fork channels: 78th Annual International Meeting, SEG, Expanded Abstracts, 27, 963-967.
- Ulrych, T., 2008, The role of amplitude and phase in processing and inversion: SEG Distinguished Lecturer, Spring 2008.